Solar Glint for Optical Ranging

Anja Brokaw, Senior Software Engineer
Cosmic AES, anja.brokaw@cosmicaes.com

Co-authors

Adrian Johnson, Principal Engineer
Cosmic AES, adrian.johnson@cosmicaes.com
Phillip Burns, Senior Research Engineer
Cosmic AES, phillip.burns@cosmicaes.com

ABSTRACT

Space has become a contested warfighting domain, and it is imperative that the Air Force can provide Space Situational Awareness (SSA) to operational commanders and commercial entities. Due to the complexity of holding custody of space objects of interest with a single sensor and to limitations in coverage areas, sensors in the Space Surveillance Network (SSN) must be tipped and cued to achieve a higher level of coverage. Exploiting existing data in new and innovative ways to tip and cue existing sensors can increase the probability of detection and characterization of space objects.

For example, as shown in Exhibit 1, an unknown orbiting object is observed by ground telescopes. From a single electro-optical sensor, it is difficult to determine the object's orbital height without additional data. A potential method is to exploit the solar glint of the object to provide additional data to assist in determining the height of the object's orbit. By calculating the point of intersection between the line-of-sight vector from a ground telescope to the orbiting object, and the Earth's cylindrical shadow at the moment an object emerges from the Earth's shadow (by observing the solar glint of the object) the accuracy of the object's geolocation is improved, thus reducing the positional uncertainty to a smaller search volume. By rapidly performing image processing and geometry calculations with existing toolsets to geolocate the object, the additional information can be used to tip the Geosynchronous Space Situational Awareness Program (GSSAP) sensors and other surveillance systems. Additionally, this solar geometry exploitation method could be used by operators to perform predictive analysis on satellite controller's intentions based on state vectors.

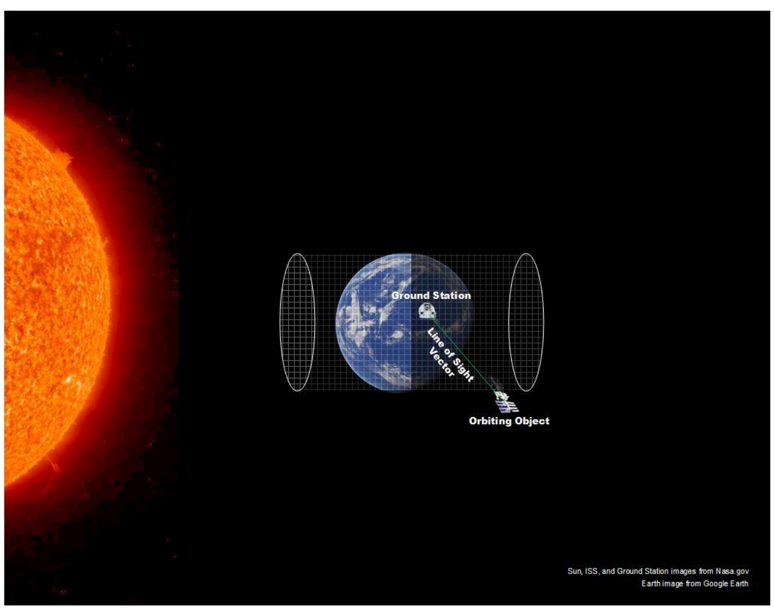

Exhibit 1: Operational Overview - object moving through Earth's shadow

TECHNICAL DESCRIPTION

General Overview

Cosmic AES suggests a method that uses the solar terminator in conjunction with ground or space-based monoscopic observations to reduce the search area for an unknown orbiting object. As shown in Exhibit 1, the Earth creates a cylindrical shadow, the geometry of which can be calculated rapidly. Using the intersection of the observing sensor's line-of-sight vector with the shadow, a more precise estimate of the position can be calculated. The refined object position can then be used to tip and cue the existing sensor network. This method might assist operators in more rapidly detecting an object's orbital decay or generating initial observations of a recently launched object.

Technical Description: Intersection of Terminator with Vector

By using ground-based optical telescopes, such as those in the Ground-based Electro-Optical Deep Space Surveillance (GEODSS) system within the SSN, those located at the U.S. Air Force Academy, or space-based sensors such as Eagle, objects in orbit can be observed. The vector $\vec{V}_{los}$ (line of sight vector from the sensor to a detected object) can be calculated using the precise angles and location ($\bar{X}_{sensor}$ = (X_s, Y_s, Z_s)) of the telescope. As shown in Exhibit 2, a ground-based optical telescope at point $\bar{X}_{sensor}$ has identified an object S along vector $\vec{V}_{los}$. The known angles of the telescope (α, β, γ) allow the calculation of the LOS vector. Although the vector can be determined, the orbiting object could be at any distance along the vector.

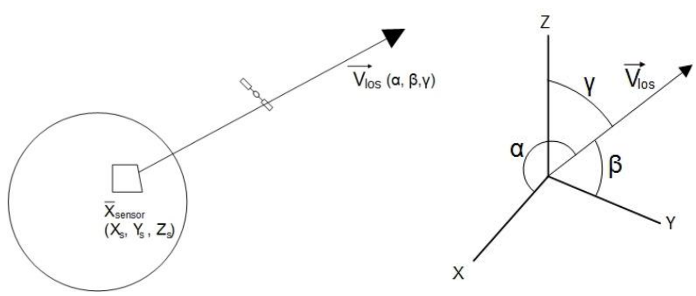

Exhibit 2: LOS Vector and resulting angles

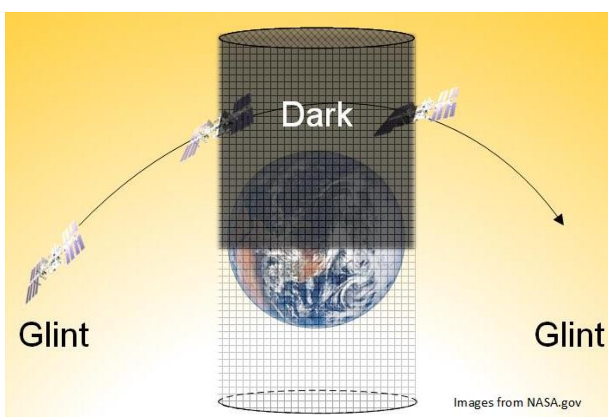

Exhibit 3: Object moving through the shadow cylinder

As the object moves through its orbit, at a certain point it disappears into the Earth's shadow as depicted in Exhibit 3. Using the solar glint reflected off the object, the point at which the object disappears or reappears from the Earth's shadow can be determined. The Earth's day/night boundary (called the solar terminator), forms a cylinder (C_{shadow}), which can be projected behind the Earth (from the Sun's perspective). The point at which the object appears from the shadow, $\bar{X}_{glint}$ = (X_g, Y_g, Z_g) can then be calculated using:

1. The angles of the telescope (α, β, γ) to determine $\vec{V}_{los}$)
2. The projected cylindrical surface (C_{shadow}) of the terminator (see exhibit 4).

The object's location in space is then determined by the intersection of a vector with the surface of a cylinder (the derivation of the equations are shown below). The calculation of the point represents the position of the object where it crosses the terminator boundary and the object's position on the $\vec{V}_{los}$ vector, providing a rough estimate of the object's geolocation in ECEF coordinate frame and effectively narrowing the search volume for other sensors.

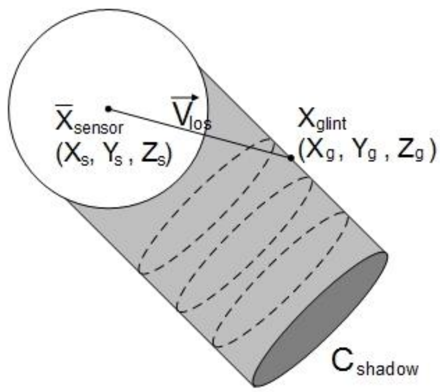

Exhibit 4: Intersection of V_{los} with cylindrical shadow

This approach to determining the height above the ellipsoid (HAE) of the orbiting object along the line-of-sight vector can be implemented using existing SSN data and does not require additional hardware.

Solar Terminator

By determining the position of the solar terminator (which can be calculated quickly and accurately for any date and time[1]), the results can then be used to determine the projection of the terminator into three-dimensional space. Using the projected terminator, the intersection between the line of sight vector $\vec{V}_{los}$ and the 3D shadow volume can then be calculated. The solar terminator model Cosmic AES employs takes into account Earth's orbital eccentricity, as well as the obliquity correction. At any given time, half of the Earth is illuminated, and the projection of the terminator on the Earth results in a nearly circular ellipse. For these calculations, the solar terminator is modeled as a circle. To further enhance the accuracy of the calculations, the elliptical model can be used; however, the complexity of the solution and resulting calculations will increase. In the scenario of using a rapidly-generated estimate of the elevation of the object to tip and cue sensors, the spherical model may suffice.

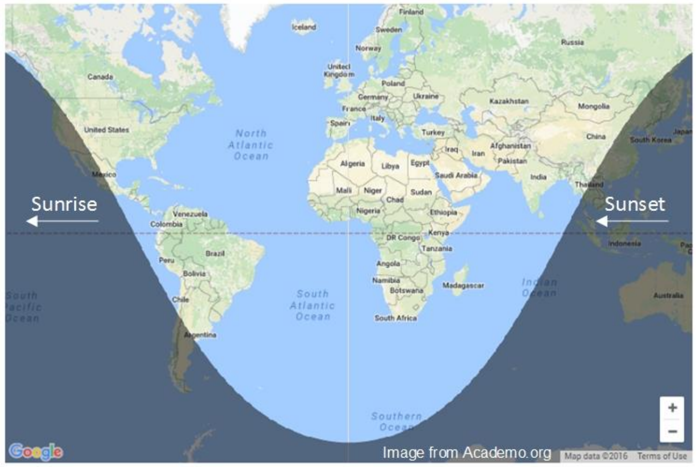

Exhibit 3: Solar terminator rendered on a Mercator projection

On a Mercator projection (Exhibit 4), the terminator appears as a sinusoid with its peaks moving between the north and south poles depending on the tilt of the Earth, and it is centered at the equator on the equinoxes. The projected cylinder behind the Earth represents the Earth's shadow, as shown in Exhibit 3. In reality, the rays of illumination from the sun are not parallel. Therefore, they create a shadow that appears as a conical volume behind the Earth. For our purposes, the solar rays are considered virtually parallel due to the size of the sun, the distance of the sun from the Earth, and the relative nearness of the orbiting object to the Earth. We can then assume that the parallel lines create a cylindrical shadow behind the Earth with a radius R equal to the radius of the Earth.

Vector/cylindEr intersection Calculation Derivation

The surface of a cylinder is defined as the set of points with a constant distance to the axis of the cylinder. If the axis of the cylinder is defined by a unit pointing vector $\hat{\mathbf{a}}$, $\mathbf{b}$ is the origin of the base of the cylinder, and $\mathbf{p}$ is an arbitrary point on the cylinder surface, then the cylinder is defined by the set of points for which the perpendicular distance to the cylinder axis is a constant r. The perpendicular distance from the point $\mathbf{p}$ to the cylinder axis can be expressed as the vector cross product of the line from the cylinder base to $\mathbf{p}$ and the vector $\mathbf{a}$ along which the cylinder axis lies. The equation of the cylinder is expressed as

$$\left\|\hat{\mathbf{a}}\times(\mathbf{p}-\mathbf{b})\right\| = r \tag{0.1}$$

The general equation of the line in space originating from point $\mathbf{x}$ one unit along a unit vector $\hat{\mathbf{n}}$ is

$$\mathbf{y} = \mathbf{x} + \hat{\mathbf{n}}\,l \tag{0.2}$$

The intersection of the line and the surface of the cylinder (if it exists) is found by substituting the expression for $\mathbf{y}$ into equation (0.1) yielding

$$\left\|\hat{\mathbf{a}}\times(\mathbf{x}+\hat{\mathbf{n}}\,l-\mathbf{b})\right\| = r \tag{0.3}$$

The subsequent algebra is simplified if equation (0.3) is rearranged as follows:

$$\left\|\hat{\mathbf{a}}\times(\hat{\mathbf{n}}\,l-\mathbf{c})\right\| = r \tag{0.4}$$

Where $\mathbf{c} = (\mathbf{b}-\mathbf{x})$. Squaring both sides of equation (0.4) results in a quadratic equation in the unknown variable l

$$\left[(\hat{\mathbf{n}}\times\hat{\mathbf{a}})\cdot(\hat{\mathbf{n}}\times\hat{\mathbf{a}})\right]l^2 - 2\left[(\mathbf{c}\times\hat{\mathbf{a}})(\hat{\mathbf{n}}\times\hat{\mathbf{a}})\right]l + (\mathbf{c}\times\hat{\mathbf{a}})\cdot(\mathbf{c}\times\hat{\mathbf{a}}) = r^2 \tag{0.5}$$

Using the quadratic formula and with help from Mathematica the solution to equation (0.5) can be shown to be

$$l = \frac{(\hat{\mathbf{n}}\times\hat{\mathbf{a}})\cdot(\mathbf{c}\times\hat{\mathbf{a}}) \pm \sqrt{(\hat{\mathbf{n}}\times\hat{\mathbf{a}})\cdot(\hat{\mathbf{n}}\times\hat{\mathbf{a}})r^2 - (\hat{\mathbf{a}}\cdot\hat{\mathbf{a}})(\mathbf{c}\cdot(\hat{\mathbf{n}}\times\hat{\mathbf{a}}))^2}}{(\hat{\mathbf{n}}\times\hat{\mathbf{a}})\cdot(\hat{\mathbf{n}}\times\hat{\mathbf{a}})} \tag{0.6}$$

If a solution to equation (0.6) exists, the position of point $\mathbf{p}$ is given as

$$\mathbf{y} = \mathbf{x} + \hat{\mathbf{n}}\,l \tag{0.7}$$

The simplified solution to equation (0.5) as expressed by equation (0.6), has been verified via Mathematica[2].

TESTING AND VALIDATION

To test and validate the proposed algorithms, a known orbiting object, such as the International Space Station (ISS), could be used for truth data. The estimated position of the ISS can be calculated using the Earth shadow algorithms with observations from ground-based optical telescopes that can follow the ISS to determine when it disappears into Earth's shadow. Position calculations would then be compared to the actual orbital data of the ISS.

SUMMARY

Determining a more accurate position of an unidentified orbiting object at a given time can make tipping and cueing other sensors for additional observations more successful. These sensors may also reveal additional characteristics by using other sensor modalities. Enabling more specific tasking for these tipped sensors will also reduce required sensor coverage time, allowing sensors to be tasked against more objects within a given time interval. This improves the opportunities for potentially discovering unknown objects, observing changes in known objects' orbits, and observing characteristics of objects.

[1] NOAA Solar Calculator: https://www.esrl.noaa.gov/gmd/grad/solcalc/
[2] https://en.wikipedia.org/wiki/User:Nominal_animal